

**NUMERICAL ANALYSIS OF OPTIMAL CONTROL  
PROBLEM GOVERNED BY TRANSIENT STOKES  
EQUATIONS WITH STATE CONSTRAINTS  
POINTWISE IN TIME**

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ABSTRACT

We consider the following optimal control problem:

$$\text{Minimize } \frac{1}{2} \int_0^T \int_{\Omega} (u(t, x) - u_d(t, x))^2 dx dt + \frac{\alpha}{2} \int_0^T \int_{\Omega} q(t, x)^2 dx dt$$

subject to transient Stokes equations

$$\begin{aligned} \partial_t u - \Delta u + \nabla p &= q && \text{in } (0, T) \times \Omega \\ \nabla \cdot u &= 0 && \text{in } (0, T) \times \Omega \\ u &= 0 && \text{on } (0, T) \times \partial\Omega \\ u(0) &= u_0 && \text{in } \Omega \end{aligned}$$

and to state constraints formulated pointwise in time, i.e.

$$\int_{\Omega} u(t, x) \cdot w(x) dx \leq b \quad \text{for all } t \in [0, T]$$

with a given  $u_d \in L^2(I; L^2(\Omega)^d)$  and  $w \in L^2(\Omega)^d$ . The domain  $\Omega \subset \mathbb{R}^d$ ,  $d = 2, 3$  is assumed to be polygonal/polyhedral and convex.

The optimality system for this problem involves a Lagrange multiplier from the space of regular Borel measures  $\mu \in \mathcal{M}([0, T])$ , which affects the regularity of the solution. We discretize the problem with inf-sup stable finite elements in space and with a discontinuous Galerkin method in time. For this discretization we provide quasi-optimal error estimates for the state and the control. The analysis is based on recently established error estimates for the Stokes system [1].

REFERENCES

- [1] N. Behringer, D. Leykekhman, B. Vexler: *Fully discrete best approximation type estimates in  $L^\infty(I; L^2(\Omega)^d)$  for finite element discretizations of the transient Stokes equations*, IMA Journal of Numerical Analysis, accepted (2022).

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