

EXTREMAL POINTS OF TOTAL GENERALIZED VARIATION BALLS IN 1D

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ABSTRACT

The total generalized variation (TGV) is a popular regularizer in inverse problems and imaging combining discontinuous solutions and higher order smoothing. In particular, empirical observations suggest that its order two version strongly favors piecewise affine functions.

We formalize this statement for the one-dimensional TGV functional by characterizing the extremal points of its sublevel sets with respect to a suitable quotient space topology. Combining them with sparsity results [2], one can prove that 1D TGV-regularized linear inverse problems with finite dimensional observations indeed admit piecewise affine minimizers.

Such a characterization also leads to precise first-order necessary optimality conditions without requiring convexity of the fidelity term, and a simple solution algorithm for TGV-regularized minimization problems.

REFERENCES

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