OPTIMAL CONTROL OF THE KIRCHHOFF EQUATION

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ABSTRACT

We consider an optimal control problem for the steady-state Kirchhoff equation, a prototype for nonlocal partial differential equations, different from fractional powers of closed operators. Existence and uniqueness of solutions of the state equation, existence of global optimal solutions, differentiability of the control-to-state map and first-order necessary optimality conditions are established. The aforementioned results require the controls to be functions in H^1 and subject to pointwise upper and lower bounds. In order to obtain the Newton differentiability of the optimality conditions, we employ a Moreau-Yosida-type penalty approach to treat the control constraints and study its convergence. The first-order optimality conditions of the regularized problems are shown to be Newton diffentiable, and a generalized Newton method is detailed. A discretization of the optimal control problem by piecewise linear finite elements is proposed and numerical results are presented.

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