

A GENERALIZED CONDITIONAL GRADIENT METHOD FOR DYNAMIC INVERSE PROBLEMS WITH OPTIMAL TRANSPORT REGULARIZATION

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ABSTRACT

We present and discuss numerical algorithms for the solution of dynamic inverse problems in which for each time point, a time-dependent linear forward operator mapping the space of measures to a time-dependent Hilbert space has to be inverted. These problems are regularized with dynamic optimal-transport energies that base on the continuity equation as well as convex functionals of Benamou–Brenier-type [1]. For the purpose of deriving properties of the solutions as well as numerical algorithms, we present sparsity results for general inverse problems that are connected with the extremal points of the Benamou–Brenier energy subject to the continuity equation. For the latter, it is proven that the extremal points are realized by point masses moving along curves with Sobolev regularity [2]. This result will be employed in numerical optimization algorithms of generalized conditional gradient type. We present instances of this algorithm that are tailored towards dynamic inverse problems associated with point tracking. Finally, the application and numerical performance of the method is demonstrated for sparse dynamic superresolution [3].

REFERENCES

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