ADAPTIVE INEXACT SMOOTHING NEWTON METHODS FOR A NONCONFORMING DISCRETIZATION OF A VARIATIONAL INEQUALITY

IBTIHEL BEN GHARBIA[§], JOELLE FERZLY[⋆,§,†,‡], MARTIN VOHRALÍK[†,‡], SOLEIMAN YOUSEF[§]

ABSTRACT

We develop an adaptive inexact smoothing Newton method for a nonconforming discretization of a variational inequality. As a model problem, we consider the contact problem between two membranes. Discretized with the finite volume method, this leads to a nonlinear algebraic system with complementarity constraints. The non differentiability of the arising nonlinear discrete problem a priori requests the use of an iterative linearization algorithm in the semismooth class like, e.g., the Newton-min [1, 2]. In this work, we rather approximate the inequality constraints by a smooth nonlinear equality, involving a positive smoothing parameter that should be drawn down to zero. This makes it possible to directly apply any standard linearization like the Newton method. The solution of the ensuing linear system is then approximated by any iterative linear algebraic solver. We carry out an a posteriori error analysis where we introduce potential reconstructions, in discrete subspaces included in $H^1(\Omega)$, as well as $\mathbf{H}(\mathrm{div}, \Omega)$-conforming discrete equilibrated flux reconstructions. With these elements, we design a posteriori estimators that provide guaranteed upper bound on the energy error between the unavailable exact solution of the continuous level and a postprocessed, discrete and available, approximation at any resolution step. It also offers a separation of the different error components, namely, discretization, smoothing, linearization, and algebraic. Moreover, we propose optimal stopping criteria and design an algorithm where all the iterative procedures (smoothing, linearization, algebraic) are adaptively stopped [3]. Finally, we confirm the performance of the proposed adaptive algorithm, in particular in comparison with the semismooth Newton method.

Keywords: elliptic variational inequality, contact problem, complementarity constraint, semismooth Newton method, smoothing Newton method, a posteriori error estimate, adaptivity, stopping criteria

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[§] IFP Energies nouvelles, 1 et 4 avenue de Bois-Préau, 92852 Rueil-Malmaison, France (ibtihel.ben-gharbia@ifpen.fr joelle.ferzly@ifpen.fr, soleiman.yousef@ifpen.fr)

[†] Inria, 2 Rue Simone Iff, 75589 Paris, France (martin.vohralik@inria.fr)

[‡] CERMICS, Ecole des Ponts , 77455 Marne-la-Vallée, France