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Workshop on Finite Element Methods and Adaptivity

Organizer: Dirk Praetorius (TU Wien)

Thursday 31.03.2022		
08:50 – 09:00	Opening	
Session 1 (Chair: Dirk Praetorius)		
09:00 – 09:40	Ernst P. Stephan (LU Hannover)	Higher-order boundary elements in the time domain: adaptivity, graded meshes and hp-versions
09:40 – 10:20	Andreas Schröder (U Salzburg)	hp-Finite elements for optimal control problems with control constraints
Coffee break		
Session 2 (Chair: Philipp Bringmann)		
11:00 – 11:40	Ngoc Tien Tran (U Jena)	Unstabilized hybrid high-order method for a class of degenerate convex minimization problems
11:40 – 12:20	Jan Valdman (U South Bohemia)	Vectorized FEM implementations in MATLAB
Lunch break		
Session 3 (Chair: Joscha Gedicke)		
14:00 – 14:40	Dietmar Gallistl (U Jena)	On the usefulness of mixed methods for eigenvalue computation
14:40 – 15:20	Sophie Puttkammer (HU Berlin)	Direct guaranteed lower eigenvalue bounds with quasi-optimal adaptive mesh-refinement
Coffee break		
Session 4 (Chair: Max Jensen)		
16:00 – 16:40	Antonio Orlando (UN Tucumán)	Compensated convex transforms computation for image processing and shape interrogation
16:40 – 17:20	Daniel Peterseim (U Augsburg)	Numerical homogenization from the finite element perspective
Wine reception (at the conference venue)		

Thursday 31.03., 09h00

Ernst P. Stephan (University of Hannover, Germany)

Higher-order boundary elements in the time domain: adaptivity, graded meshes and hp-versions

Abstract. We discuss recent work with A. Aimi, G. Di Credico, H. Gimperlein and C. Oezdemir on the hp and adaptive approximations of solutions to the time-dependent wave and Lame equations by boundary elements. Of particular interest are problems in polyhedral domains or outside a screen, where the solution exhibits singularities at the edges and corners. Based on ideas by Plamenevskii and coauthors, we consider the best approximation by piecewise polynomials of tensor product type in space and time. Quasi-optimal approximation rates are presented for the hp method on quasi-uniform or algebraically graded meshes. In particular the p-version converges at twice the rate of the h-version, as in the time-independent case. Furthermore, an a posteriori error estimate is presented, which gives rise to adaptive mesh refinement procedures. The adaptive procedure recovers the convergence rates known for time-independent problems. Numerical experiments illustrate the theory.

Thursday 31.03, 09h40

Andreas Schröder (University of Salzburg, Austria)

hp-Finite elements for optimal control problems with control constraints

Abstract. In this talk a distributed elliptic control problem with control constraints is considered which is formulated as a three field problem and consists of two variational equations for the state and the co-state variables as well as of a variational inequality for the control variable. Each of the three variables are discretized independently by hp-finite elements. Sufficient conditions for the unique existence of a discrete solution are stated and a priori error estimates are presented in terms of the mesh size as well as of the polynomial degree. Moreover, reliable and efficient a posteriori error estimates are discussed, which enables h- and hp-adaptive mesh refinements. Several numerical experiments demonstrate the applicability of the discretization with hp-finite elements and of the a posteriori error estimates.

Thursday 31.03., 11h00

Ngoc Tien Tran (University of Jena, Germany)

Unstabilized hybrid high-order method for a class of degenerate convex minimization problems

Abstract. The relaxation in the calculus of variation motivates the numerical analysis of a class of degenerate convex minimization problems with non-strictly convex energy densities with some convexity control and two-sided p-growth. The minimizers may be non-unique in the primal variable but lead to a unique stress. The approximation by hybrid high-order methods utilizes a reconstruction of the gradients with piecewise Raviart-Thomas finite elements without stabilization on a regular triangulation into simplices. The application of this HHO method to the class of degenerate convex minimization problems allows for a unique H(div) conforming stress approximation. This allows for a priori and a posteriori error estimates for the stress error in Lebesgue norms and a computable lower energy bound.

Thursday 31.03., 11h40

Jan Valdman (University of South Bohemia, Czech Republic)

Vectorized FEM implementations in MATLAB

Abstract. Following the popular implementation article of Carsten Carstensen et al. from 1999 on “Remarks around 50 lines of Matlab: short finite element implementation”, we show some own development of vectorization tools for FEM assemblies in MATLAB. The focus will be given to nodal and edge elements as well as C1 elements. In addition to implementation details, the resulting papers exploiting these implementations will be surveyed as well. At last a partly automated technique for solving energy minimization problems will be explained.

Thursday 31.03., 14h00

Dietmar Gallistl (University of Jena, Germany)

On the usefulness of mixed methods for eigenvalue computation

Abstract. By introducing a dual (or stress) variable, symmetric positive definite PDE eigenvalue problems can be posed in an equivalent mixed formulation. Such problems are formulated as saddle-point systems, which implies a higher computational cost compared with the original problem. While application-related advantages of operating with the mechanically relevant stress variable are sometimes mentioned for justifying and advertising mixed methods, a decisive structural advantage for systematically using mixed methods in eigenvalue computations has remained obscure. It is the aim of this contribution to reveal a basic and rather generic feature of dual mixed formulations and related finite element discretizations that allows for the computation of guaranteed lower eigenvalue bounds in many practical examples and where the involved constants are related to properties of the underlying PDE operator rather than special properties of the discretization space (as it is the case in the pioneering contributions from the past decade). The applications presented in this paper include the Laplacian, general second-order scalar coercive operators, the Lamé eigenvalues of linear elasticity, the Steklov eigenvalue problem, the Stokes system, and the biharmonic eigenvalue problem.

Thursday 31.03., 14h40

Sophie Puttkammer (HU Berlin, Germany)

Direct guaranteed lower eigenvalue bounds with quasi-optimal adaptive mesh-refinement

Abstract. Guaranteed lower eigenvalue bounds (GLBs) for elliptic PDE eigenvalue problems are of high relevance in theory and praxis, e.g., in a safety analysis in computational mechanics, for the detection of spectral gaps, or for valid bounds of the continuity constants of the Sobolev embedding. Due to the Rayleigh-Ritz (or) min-max principle conforming finite element methods (FEMs) provide guaranteed upper eigenvalue bounds. A post-processing for nonconforming FEMs computes GLBs. However, the maximal mesh-size enters as a global parameter in the GLB and may cause significant underestimation for adaptive mesh-refinement.

New skeletal and hybrid high-order (HHO) methods with a fine-tuned stabilization lead to direct GLBs. In other words, a specific smallness assumption on the maximal mesh-size makes the computed k-th discrete eigenvalue a GLB for the k-th Dirichlet eigenvalue. For the m-Laplace operator a recently introduced extra-stabilized nonconforming Crouzeix-Raviart (m = 1) or Morley (m = 2) finite element eigensolver computes GLBs as well. This talk shows striking numerical evidence for the superiority of a new adaptive eigensolver that motivates the convergence analysis. For the extra-stabilized nonconforming methods (a generalization of) known abstract arguments entitled as the axioms of adaptivity verify the convergence of the GLB towards a simple eigenvalue with optimal rates.

Thursday 31.03., 16h00

Antonio Orlando (National University of Tucumán, Argentine)

Compensated convex transforms computation for image processing and shape interrogation

Abstract. Compensated convex transforms are a class of geometric convexity-based transforms that enjoy tight-approximation and locality properties, and Hausdorff stability. Such properties can be exploited to develop novel multiscale, parametrized, and robust methods for smoothing functions, identifying singularities in functions, and approximating sampled functions in Euclidean spaces.

After reviewing definitions, basic properties, and different equivalent characterizations, I will introduce a linear–time algorithm that computes the compensated convex transforms. The algorithm relies on the characterization of the compensated convex transforms as proximal hull and uses its definition in terms of the Moreau envelopes. The algorithm computes in fact the Moreau envelope of a function and is useful on its right for general applications of the Moreau envelope. An estimate of the iterations needed for computing the exact discrete Moreau envelope will also be given.

I will present prototype examples with analytical expressions of the transforms and illustrative numerical examples for the detection of features in image or data, multiscale medial axis extraction, surfaces intersections, surface reconstruction from level sets and point cloud, approximation of scattered data, and noise removal from images, and image inpainting. The numerical examples show the performance of this new class of methods compared to the state-of-the-art ones.

This is joint work with Kewei Zhang (University of Nottingham, UK) and Elaine Crooks (Swansea University, UK)

Thursday 31.03., 16h40

Daniel Peterseim (University of Augsburg, Germany)

Numerical homogenization from the finite element perspective

Abstract. Numerical homogenization is a methodology for the computational solution of multiscale partial differential equations. It aims at reducing complex large-scale problems to simplified numerical models valid on some target scale of interest, thereby accounting for the impact of features on smaller scales that are otherwise not resolved. This talk reviews a central approach in this context and discusses the intended and unintended influence that Carsten Carstensen has had on its development.

Friday 01.04.2022		
Session 5 (Chair: Ernst P. Stephan)		
09:00 – 09:40	Dirk Praetorius (TU Wien)	On optimal computational costs of AFEM
09:40 – 10:20	Philipp Bringmann (HU Berlin)	Convergence analysis and numerical comparison of adaptive least-squares finite element methods
Coffee break		
Session 6 (Chair: Andreas Schröder)		
11:00 – 11:40	Lukas Gehring (U Jena)	An initial refinement is not necessary for #Bisections <= C * #Markings in any dimension
11:40 – 12:20	Johannes Storn (U Bielefeld)	H1 stability of the L2 projection and its relation to mesh grading
Lunch break		
Session 7 (Chair: Daniel Peterseim)		
14:00 – 14:40	Max Jensen (U Sussex)	Finite element approximation of Hamilton-Jacobi-Bellman equations with nonlinear mixed boundary conditions
14:40 – 15:20	Joscha Gedicke (U Bonn)	A posteriori error analysis for symmetric mixed Arnold-Winther FEM
Coffee break		
Session 8 (Chair: Dietmar Gallistl)		
16:00 – 16:40	Neela Nataraj (IIT Bombay)	Lowest-order equivalent nonstandard finite element methods for biharmonic plates
16:40 – 17:20	Carsten Carstensen (HU Berlin)	Three decades of the Crouzeix-Falk conjecture and five decades of Crouzeix-Raviart finite elements
17:20 – 17:30	Closing	
Conference Dinner (starting 19:00)		

Friday 01.04., 09h00

Dirk Praetorius (TU Wien, Austria)

On optimal computational costs of AFEM

Abstract. Meanwhile the mathematical understanding of optimal convergence of adaptive algorithms has matured. Unlike the seminal works on adaptive adaptive wavelet methods, however, most of the works on adaptive standard FEM consider optimal rates with respect to the number of degrees of freedom, i.e., the focus is on error/estimator vs. dimension of the FEM space. Practically, the focus should rather be on error/estimator vs. overall computational cost/time. In our presentation, we explain the difference of both concepts and then show how optimal convergence with respect to computational costs can mathematically be proved. As we will see, key argument is the notion of full linear convergence.

The talk is mainly based on joint work with Roland Becker (University of Pau, France), Maximilian Brunner (TU Wien), Gregor Gantner (University of Amsterdam, The Netherlands), and Michael Innerberger (TU Wien).

Friday 01.04., 09h40

Philipp Bringmann (HU Berlin, Germany)

Convergence analysis and numerical comparison of adaptive least-squares finite element methods

Abstract. Due to the built-in a posteriori error control, the least-squares finite element methods (LSFEMs) are a favourable choice for adaptive mesh-refining algorithms. This talk gives an overview of the convergence results for adaptive LSFEMs in the literature. First, the built-in error estimator leads to Q-linear convergence in an adaptive algorithm with collective marking. Second, an alternative residual-based error estimator and a separate marking strategy with data approximation even guarantee optimal convergence rates for the error in the natural underlying norm. Third, collective marking with the alternative error estimator provides optimal convergence rates for a weaker norm. An experimental comparison of all three adaptive algorithms confirms these findings. This talk investigates the choice of the parameters in the marking and refinement strategies as well as the performance of the adaptive algorithms.

Friday 01.04., 11h00

Lukas Gehring (University of Jena, Germany)

An initial refinement is not necessary for #Bisections $\leq C\cdot$ #Markings in any dimension

Abstract. A triangulation of a polytope into simplices is refined recursively by standard bisection. In every refinement round, some simplices which have been marked by an external algorithm are bisected and some others around also must be bisected to retain regularity of the triangulation. Assuming a certain initial condition, Binev, Dahmen, DeVore (2D) [Numer. Math., 97(2004), 219–268] and Stevenson (nD) [Math. of Comp. 77(2008), 227-241] bounded the final number of simplices at the end of the iterated refinement process by the initial number of simplices plus a multiple of the total number of marked simplices. A 3D triangulation is presented that cannot be renumbered satisfying this initial condition. The final number of simplices is also bounded by a linear combination of the initial number of simplices and the number of marked simplices for the renumbering of Alkämper, Gaspoz and Klöfkorn [SIAM J. Sc. Comp. 40/6(2018) A3853-A3872], fulfilling a weaker initial condition. Supposably, this is the first Binev–Dahmen–DeVore theorem for an arbitrary n-dimensional initial triangulation without an initial refinement.

Friday 01.04., 11h40

Johannes Storn (University of Bielefeld, Germany)

H^1 stability of the L^2 projection and its relation to mesh grading

Abstract. The L^2-projection mapping to Lagrange finite element spaces is an important tool in numerical analysis. For adaptively generated meshes the proof of Sobolev stability is challenging and assumptions on the mesh grading are unavoidable. We present recent results on L^p and $W^{1,p}$-stability in 2D and 3D for any polynomial degree and for any space dimension under suitable conditions on the mesh grading. We further discuss the mesh grading and give an outlook on ongoing research concerning mesh gradings for meshes generated by the Maubach-Traxler algorithm.

This is joint work with Lars Diening and Tabea Tscherpel from Bielefeld University (Germany) and Fernando Gaspoz from Universidad Nacional del Litoral (Argentina).

Friday 01.04., 14h00

Max Jensen (University of Sussex, UK)

Finite element approximation of Hamilton-Jacobi-Bellman equations with nonlinear mixed boundary conditions

Abstract. We show uniform convergence of monotone P1 finite element methods to the viscosity solution of isotropic parabolic Hamilton-Jacobi-Bellman equations with mixed boundary conditions on unstructured meshes and for possibly degenerate diffusions. Boundary operators can generally be discontinuous across face-boundaries and type changes. Robin-type boundary conditions are discretised via a lower Dini derivative. In time the Bellman equation is approximated through IMEX schemes. Existence and uniqueness of numerical solutions follows through Howard's algorithm. We show how equations of this type naturally appear in models of mathematical finance.

Friday 01.04., 14h40

Joscha Gedicke (University of Bonn, Germany)

A posteriori error analysis for symmetric mixed Arnold-Winther FEM

Abstract. The development of mixed finite element methods for linear elasticity with strongly imposed symmetry has been a long standing problem until the beginning of this century. Surprisingly for a mixed method, nodal stress degrees of freedom are necessary in order to fulfill the strong symmetry. This interesting mixed finite element also poses some difficulties for the derivation of residual based a posteriori error estimators. In a first attempt we make use of the residual a posteriori error estimator techniques for weakly symmetric stresses introducing an auxiliary approximation of the skew-symmetric gradient via a postprocessing. The second version then makes fully use of the imposed symmetry of the stress approximations utilising integration by parts twice and a suitable decomposition into tangential-tangential and normal-normal parts, similarly to the residual a posteriori error analysis for plate problems.

Friday 01.04., 16h00

Neela Nataraj (IIT Bombay, India)

Lowest-order equivalent nonstandard finite element methods for biharmonic plates

Abstract. The popular (piecewise) quadratic schemes for the biharmonic equation based on triangles are the nonconforming Morley finite element, the discontinuous Galerkin, the C^0 interior penalty, and the WOPSIP schemes. Those methods are modified in their right-hand side $F \in H^{-2}(\Omega)$ replaced by $F(JI_M)$ and then are quasi-optimal in their respective discrete norms. The smoother JI_M is defined for a piecewise smooth input function by a (generalized) Morley interpolation I_N followed by a companion operator J. An abstract framework for the error analysis in the energy, weaker and piecewise Sobolev norms for the schemes is outlined and applied to the biharmonic equation.

Friday 01.04., 16h40

Carsten Carstensen (HU Berlin, Germany)

Three decades of the Crouzeix-Falk conjecture and five decades of Crouzeix-Raviart finite elements

Abstract. Invented in 1973 for any polynomial degree to solve the Stokes equations with piecewise divergence-free velocities, the Crouzeix-Raviart finite elements allowed various applications to nonlinear PDE over the years. In their lowest-order version, they compete with conforming schemes in elasticity and the Bingham flow. But, most unexpectedly, they allow guaranteed lower eigenvalue bounds for the Laplacian and guaranteed lower energy bounds in convex minimization; they even overcome the Lavrentiev phenomenon.

The presentation discusses contributions to the aforementioned model problems from the speaker's group and former students. The circle through all those topics will be closed with remarks on the very beginning of the Crouzeix-Raviart finite element history: Crouzeix-Raviart triangular elements are inf-sup stable for any polynomial degree provided the mesh has interior vertices. This answers a conjecture of M. Crouzeix and R. Falk after three decades.

The time restrictions do not permit any discussion of a medius analysis, as addressed e.g. in an overview paper by S. Brenner, or of companion operators. Only minimal comments on the a posteriori error control and the mathematics of adaptive mesh-refining algorithms might be expected.