Multi-scale FEM for the eddy current problem in laminated media

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***Abstract*—The simulation of eddy currents in laminated iron cores by the finite element method (FEM) is of great interest in the design of electrical machines and transformers. The overall dimensions of an iron core and the thickness of the laminates are very different. A finite element model which considers each laminate requires many finite elements (FEs) leading to extremely large systems of equations and prohibitively high computational costs. Therefore, an improved multi-scale FEM to resolve this problem is studied. A numerical example demostrates the accuracy and the low computational costs.**

***Index Terms*—Eddy currents, multi-scale finite element methods, laminates, numerical simulation.**

I. Introduction

To improve the local approximation the magnetic flux density parallel to the lamination is expanded into orthogonal even polynomials, so-called skin effect sub-basis functions, in [1] and higher order corrector terms were determined solving the associated cell problems in [2]. To improve our multi-scale finite element method (MSFEM) approach used in [3] we propose an extension presented below.

II. Numerical Method and Example

The eddy current problem to be solved is sketched in Fig. 1. It consists of a laminated material Ω_m enclosed by air Ω_0, i.e., the entire domain $\Omega = \Omega_m \cup \Omega_0$, with an outer boundary Γ and normal vector $\boldsymbol{n}$. The parameters are the magnetic permeability μ, the electric conductivity σ, which is zero in air and the angular frequency ω. The eddy current boundary value problem with the magnetic vector potential $\boldsymbol{A}$ in the frequency domain reads as

$$\operatorname{curl} \frac{1}{\mu} \operatorname{curl} \boldsymbol{A} + j\omega\sigma\boldsymbol{A} = \boldsymbol{0} \quad \text{in } \Omega \subset \mathbb{R}^2, \tag{1}$$

$$\boldsymbol{A} \times \boldsymbol{n} = \boldsymbol{\alpha} \quad \text{on } \Gamma. \tag{2}$$

The standard polynomial basis of the FEs [4] are augmented by periodic orthogonal micro-shape functions as shown in Fig. 2. Thus, the multi-scale approach

$$\tilde{\boldsymbol{A}} = \boldsymbol{A}_0 + \phi_1 \begin{pmatrix} 0 \\ A_1 \end{pmatrix} + \phi_3 \begin{pmatrix} 0 \\ A_3 \end{pmatrix} + \nabla(\phi_1 w_1) + \nabla(\phi_3 w_3) \tag{3}$$

for a two dimensional problem with the magnetic flux perpendicular to the plane of projection has been assumed. The main magnetic field is an even function across a laminate. Consequently, the variation of the current density and that of $\boldsymbol{A}$ is mainly an odd function. Thus, it suffices to consider only odd micro-shape functions in (3).

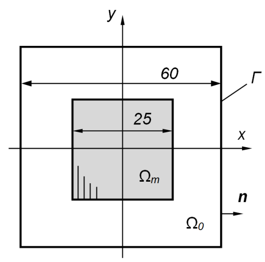

Fig. 1. Eddy current problem (dimensions in mm).

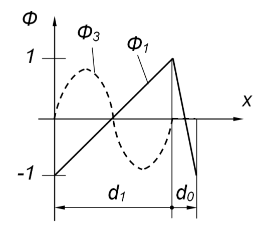

Fig. 2. Orthogonal micro-shape functions.

The iron stack consists of 100 laminates. A thickness of both layers, iron and air, of $d_1 + d_0 = 0.25$mm, a fill factor of $c_f = 0.9$, a conductivity of $\sigma = 2 \cdot 10^6$S/m and relative permeability of $\mu_r = 50,000$ were selected. The reference solution using a FE model where each laminate is considered has been computed very accurately. One FE layer was used across the thickness of the iron layer. The number of degrees of freedom equals to $123,564$ for 2^{nd} order FEs. The FE order was 6 for the smallest penetration depths. The FE model of MSFEM consists of $3,586$ FEs for all simulations. A comparison of the accuracy obtained by (3) with that [3] where only ϕ_1 is used is shown in Fig. 3. The penetration depth was varied by the frequency. The accuracy of MSFEM with (3) is excellent.

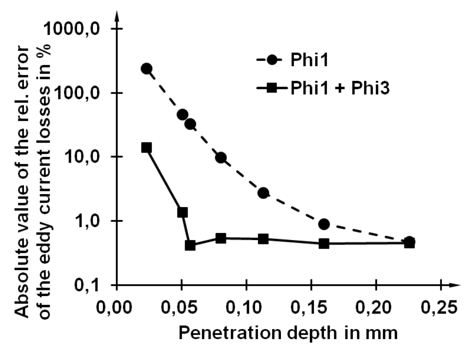

Fig. 3. Absolute value of the relative error of the eddy current losses with respect to the penetration depth.

References

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