An Efficient Reformulation of a Multiscale Method for the Eddy Current Problem

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***Abstract*—The performance of classical multi-scale methods for the 2D $\mathrm{H(curl)}$ eddy current problem in layered materials is studied for layer widths approaching zero. These results are compared to a new more efficient reformulation which does not have to utilize additional spaces.**

***Index Terms*— eddy current, multiscale, reformulation**

I. Problem Setting

Consider the eddy current problem

$$\int_\Omega \mu^{-1}\mathrm{curl}\mathbf{A}\ \mathrm{curl}\mathbf{v} + j\omega\sigma\mathbf{A}\mathbf{v}\ d\Omega = \int_\Omega \mathbf{J}\mathbf{v} \quad \forall \mathbf{v} \in \mathrm{H(curl)}. \tag{1}$$

Let the domain $\Omega \subset \mathbb{R}^d$ be composed of an outer air domain Ω_0 surrounding a layered material Ω_m consisting of e.g. iron sheets of width d_1 separated by air gaps of width d_2, see Fig. 1.

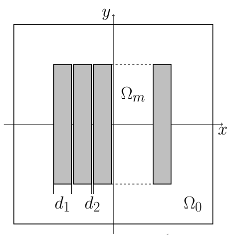

Fig. 1. The layered domain Ω.

II. Method and Reformulation

In [1] a multiscale ansatz for (1) of the form

$$\mathbf{A} = \mathbf{A}_0 + \phi \begin{pmatrix} 0 \\ A_1 \end{pmatrix} + \nabla(\phi w) \tag{2}$$

is developed, with a piecewise linear micro-shape function ϕ. $A_0, A_1, w \in \mathrm{V} := \mathrm{H(curl)} \times \mathrm{L}^2 \times \mathrm{H}^1$ are the unknown functions. Using this ansatz in (1) yields a new formulation of the form

$$\int_\Omega a\left(\begin{pmatrix} \mathbf{A}_0 \\ A_1 \\ w \end{pmatrix}, \begin{pmatrix} \mathbf{v}_0 \\ v_1 \\ q \end{pmatrix}\right)\ d\Omega = \int_\Omega \mathbf{J}\mathbf{v}_0\ d\Omega \quad \forall \begin{pmatrix} \mathbf{v}_0 \\ v_1 \\ q \end{pmatrix} \in \mathrm{V}. \tag{3}$$

with a bilinearform a involving averaged coefficients as described in [1].

Using specific test functions in (3) and incorporating properties of the solution yields additional identities for the unknown functions, which allow the expressions

$$A_1 = C_1 curl\mathbf{A}_0 + C_2 \frac{\partial}{\partial y}(\mathbf{A}_0)_1, \quad w = C_3(\mathbf{A}_0)_1 \tag{4}$$

with the constants C_1, C_2, C_3 depending on the problem parameters and $(\mathbf{A}_0)_1$ being the first component of $\mathbf{A}_0$. Using these identities, (3) can be reformulated as

$$\int_\Omega \tilde{a}(\mathbf{A}_0, \mathbf{v}_0)\ d\Omega = \int_\Omega \mathbf{J}\mathbf{v}_0\ d\Omega. \quad \forall \mathbf{v}_0 \in \mathrm{H(curl)} \tag{5}$$

where the additional FEM spaces have been eliminated. The formulation (5) assumes that the FEM implementation of $\mathrm{H(curl)}$ allows for gradient evaluations on each element.

III. Numerical Results

Figure 2 shows the errors of both the standard multiscale solution and the reformulation in the L^2-norm weighted by the electric conductivity σ and in the $\mathrm{H(curl)}$ energy norm weighted by the magnetic reluctivity μ^{-1}.

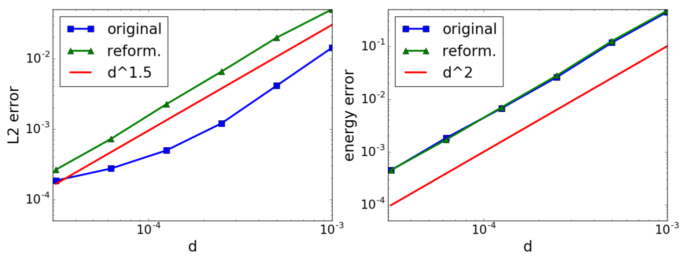

Fig. 2. Errors of the original and reformulated multi-scale solution at different sheet widths $d = d_1 + d_2$.

In both cases the reformulation shows the same rate of convergence for decreasing iron sheet widths. While the error in the weighted energy norm is nearly identical, there is a significant discrepancy in the L^2-norm for larger values of d, which vanishes as the sheet width approaches 0.

IV. Acknowledgment

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References

[1] K. Hollaus and J. Schöberl, "Homogenization of the eddy current problem in 2d," ser. 14th Int. IGTE Symp., Graz, Austria, Sep. 2010, pp. 154–159.