Multi-Scale FEM and Magnetic Vector Potential $A$ for 3D Eddy Currents in Laminated Media

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Abstract—The simulation of eddy current losses by the finite element method in laminated cores of electrical devices is still a challenging task. The dimensions of the iron core and the thickness of the laminates are very different. Thus, finite element models considering each laminate require many finite elements leading to extremely large systems of equations. A multi-scale finite element method with the magnetic vector potential has been developed to cope with 3D problems considering edge effects. Numerical simulations demonstrate a remarkable accuracy and low computational costs.

Index Terms—Laminated media, magnetic vector potential, multi-scale FEM, three dimensional eddy currents.

I. INTRODUCTION

In the multi-scale finite element method (MSFEM) proposed in [1] the main magnetic flux density parallel to the lamination is expanded by so called skin effect sub-basis functions neglecting edge effects. In this work, the MSFEM in [2] based on the magnetic vector potential $A$ is extended appropriately into three dimensions considering also edge effects.

II. EDdy CURRENT PROBLEM

A. Boundary Value Problem in the Frequency Domain

The eddy current problem to be solved consists of a conducting material $\Omega_c$ enclosed by air $\Omega_0$, i.e., $\Omega = \Omega_c \cup \Omega_0$ with an outer boundary $\Gamma$. The boundary value problem in the frequency domain with $A$ reads as

$$\begin{align*}
\frac{1}{\mu} \text{curl} A + j\omega \sigma A &= 0 \quad \text{in } \Omega \subset \mathbb{R}^3 \\
A \times n &= \alpha \quad \text{on } \Gamma,
\end{align*}$$

(1)

(2)

where $\mu$ is the magnetic permeability, $\sigma$ is the electric conductivity, $\omega$ is the angular frequency and $j$ is the imaginary unit. Dirichlet boundary conditions are prescribed on $\Gamma$.

B. Multi-Scale Approach

The feasible three-dimensional multi-scale approach

$$\tilde{A} = A_0 + \phi(0,A_{12},A_{13})^T + w(\phi_x,0,0)^T$$

(3)

with respect to Cartesian coordinates, where the normal vector of the lamination points in $x$-direction has been assumed. The quantities $A_0$, $A_{12}$ and $A_{13}$ in (3) stand for the mean value and for scalar components of the magnetic vector potential. The last term in (3) taking into account of edge effects consists of the scalar function $w_1$ and the derivative $\phi_x$ with respect to $x$ of the periodic micro-shape function $\phi$. The micro-shape function considers the periodic structure of the laminated iron in Figure 1.

III. NUMERICAL EXAMPLE

The laminated stack consists of 100 laminates, see Figure 1. A thickness of both, iron layer and air gap, of $d + d_0 = 0.25\text{mm}$, an unfavorable fill factor of $c_f = 0.9$, a conductivity of $\sigma = 2 \cdot 10^6 \text{S/m}$, a frequency of $f = 50\text{Hz}$ and a relative permeability of $\mu_r = 50,000$ were selected.

A comparison of the eddy current losses and the required computer resources are summarized in Table I.

Table I: Numerical Data

<table>
<thead>
<tr>
<th>Method</th>
<th>Losses in W</th>
<th>No. Unknowns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Solution</td>
<td>1.789</td>
<td>4,442,592</td>
</tr>
<tr>
<td>MSFEM</td>
<td>1.744</td>
<td>388,889</td>
</tr>
</tbody>
</table>

Figure 1: Laminated iron with dimensions in mm.

ACKNOWLEDGMENT

This work was supported by the Austrian Science Fund (FWF) under Project P 27028-N15.

REFERENCES